① a $5^0 + 5^1 + 5^2 = 1 + 5 + 25 = 31$

b $3^0 + 4^1 + 5^2 = 1 + 4 + 25 = 30$

c $2^0 + 2^1 + 2^2 = 1 + 2 + 4 = 7$

d $1^0 + 2^1 + 3^2 = 1 + 2 + 9 = 12$

e $7^0 + 8^1 + 9^2 = 1 + 8 + 81 = 90$

f $11^0 + 11^1 + 11^2 = 1 + 11 + 121 = 133$

g $9^0 + 10^1 + 11^2 = 1 + 10 + 121 = 132$

h $2^0 + 4^2 + 5^3 = 1 + 16 + 125 = 142$

② a $12^0 \times 7 = 7$

b $8^0 \times 3 = 3$

c $7^0 \times 4 = 4$

d $9^0 \times 11 = 11$

e $14^0 \times 3 = 3$

③ $8^2 = 4^3$ as $8^2 = 64$ and $4^3 = 64$

④ a $m^3 \times m^2 \times m = m^6$

b $2m^3 \times 5m^2 \times m = 10m^6$

c $6m^3 \times m^2 \times 4m = 24m^6$

d $2m^7 \times 2m^5 \times 3m^3 = 12m^{15}$

e $m^5 \times m = m^6$

⑤ a $\dfrac{x^5}{x^2} = x^{5-2} = x^3$

b $\dfrac{x^7}{x^5} = x^{7-5} = x^2$

c $\dfrac{x^{11}}{x^{-3}} = x^{11-(-3)} = x^{11+3} = x^{14}$

d. $\dfrac{x^{-6}}{x^4} = x^{-6-4} = x^{-10}$

e. $\dfrac{x^{11}}{x^{13}} = x^{11-13} = x^{-2}$

⑥ a $(5^2)^3 = 5^{2\times3} = 5^6$

b $(3^3)^6 = 3^{3\times6} = 3^{18}$

c $(4^4)^4 = 4^{4\times4} = 4^{16}$

d $(6^5)^5 = 6^{5\times5} = 6^{25}$

e $(3^2)^3 = 3^{2\times3} = 3^6$

f $(2^5)^5 = 2^{5\times5} = 2^{25}$

g $(-1^1)^2 = (-1)^{1\times2} = (-1)^2 = 1$

h $(-2^2)^5 = (-2)^{10}$

i $(3^4)^2 = 3^{4\times2} = 3^8$

⑦ $\dfrac{2^3 \times 2^7}{2^8} = 2^{3+7-8} = 2^2 = 4$

⑧ a $\dfrac{2\times4\times8}{64\times16} = \dfrac{2^1\times2^2\times2^3}{2^6\times2^4} = 2^{1+2+3-6-4} = 2^{-4} = 16^{-1} = \dfrac{1}{16}$

b $\dfrac{16\times32\times128}{64\times256} = \dfrac{2^4\times2^5\times2^7}{2^6\times2^8} = 2^{4+5+7-6-8} = 2^2 = 4$

c $\dfrac{2^4\times2^5\times2^7}{2^6\times2^8} = 2^{4+5+7-6-8} = 2^2 = 4$

d $\dfrac{(2^4)^3\times(2^5)^6\times2^7}{(2^6)^2\times(2^8)^3} = \dfrac{2^{12}\times2^{30}\times2^7}{2^{12}\times2^{24}} = 2^{12+30+7-12-24} = 2^{13} = 8192$

e $\dfrac{(3^4)^3\times(3^5)^6\times3^7}{(3^6)^2\times(3^8)^3} = \dfrac{3^{12}\times3^{30}\times3^7}{3^{12}\times3^{24}} = 3^{12+30+7-12-24} = 3^{13} = 1,594,323$

f. $\dfrac{512^2\times64^3\times256^5}{1024^3\times32^7} = \dfrac{(2^9)^2\times(2^6)^3\times(2^8)^5}{(2^{10})^3\times(2^5)^7} = \dfrac{2^{18}\times2^{30}\times2^{40}}{2^{30}\times2^{35}}$

$= 2^{18+30+40-30-35} = 2^{13} = 8192$

9 Express as single powers

a $\dfrac{x^3 \times x^7 \times x^4}{x^5} = x^{3+7+4-5} = x^9$

b $x^{\frac{1}{2}} \times x^{\frac{3}{4}} = x^{2/4} \times x^{3/4} = x^{5/4}$

c $\sqrt{x} \times x^{\frac{5}{2}} = x^{\frac{1}{2}} \times x^{\frac{5}{2}} = x^{6/2} = x^3$.

10

$x^{\frac{3}{2}} = 8$

$\therefore \left(x^{3/2}\right)^2 = 8^2$

$\therefore x^3 = 64$

$\therefore x = 4$

$x^{-\frac{1}{3}} = 10$

$\therefore \left(x^{-\frac{1}{3}}\right)^{-1} = 10^{-1}$

$\therefore x^{\frac{1}{3}} = 10^{-1}$

$\therefore \left(x^{\frac{1}{3}}\right)^3 = \left(10^{-1}\right)^3$

$x = 10^{-3}$

$= \dfrac{1}{1000}$

$4^x = 2^x + 56$

As $4^x = 2^x \times 2^x = (2^x)^2$

$(2^x)^2 = 2^x + 56$

Let $y = 2^x$

So $y^2 = y + 56$

$\therefore y^2 - y - 56 = 0$

$\therefore (y-8)(y+7) = 0$

$y - 8 = 0$ $\quad$ $y + 7 = 0$

$\therefore y = 8$ $\quad$ $\therefore y = -7$

As $y = 2^x$

$2^x = 8$ or $2^x = -7$

$2^x > 0$ for all x,

$2^x = 8$

$\therefore x = 3$ as $2^3 = 8$

11 $\left(3^{\frac{7}{2}}-3^{\frac{1}{2}}\right)^2=\left(3^{\frac{7}{2}}-3^{\frac{1}{2}}\right)\left(3^{\frac{7}{2}}-3^{\frac{1}{2}}\right)$

	$3^{\frac{7}{2}}$	$-3^{\frac{1}{2}}$
$3^{\frac{7}{2}}$	3^7	-3^4
$-3^{\frac{1}{2}}$	-3^4	$+3$

$=3^7-3^4-3^4+3$

$=2187-81-81+3$

$=2028$

12 $x-\sqrt{x}-6=0$

Let $y=\sqrt{x}$

So $y^2-y-6=0$

	y	-3
y	y^2	$-3y$
$+2$	$+2y$	-6

$(y+2)(y-3)=0$

$y+2=0$ $\quad y-3=0$

$y=-2$ $\quad y=3$

As $y=\sqrt{x}$

The only valid solution in this is $y=3$ as $\sqrt{x}>0$

So we have:

$y=3$

$\therefore \sqrt{x}=3$

$\therefore (\sqrt{x})^2=3^2$

$\therefore x=9$

13 $(4x+\sqrt{3})^5$

$$\begin{array}{ccccccccccc} & & & & & 1 & & & & & \\ & & & & 1 & & 1 & & & & \\ & & & 1 & & 2 & & 1 & & & \\ & & 1 & & 3 & & 3 & & 1 & & \\ & 1 & & 4 & & 6 & & 4 & & 1 & \\ 1 & & 5 & & 10 & & 10 & & 5 & & 1 \end{array}$$

$$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

So we have $a = 4x$, $b = \sqrt{3}$

$$(4x+\sqrt{3})^5 = (4x)^5 + (4x)^4(\sqrt{3}) + (4x)^3(\sqrt{3})^2 + (4x)^2(\sqrt{3})^3 + (4x)(\sqrt{3})^4 + (\sqrt{3})^5$$

$$= 1024x^5 + 256\sqrt{3}x^4 + 192x^3 + 48\sqrt{3}x^2 + 36x + 9\sqrt{3}$$

14 $\dfrac{3}{\sqrt{x}} + \dfrac{2}{x} = 5$

Let $y = \dfrac{1}{\sqrt{x}}$

So $3y + 2y^2 = 5$

$\therefore$ $2y^2 + 3y - 5 = 0$

Using the formula

$$y = \frac{-3 \pm \sqrt{9 - 4(2)(-5)}}{2(2)}$$

$$= \frac{-3 \pm \sqrt{49}}{4}$$

$\therefore$ $y = \dfrac{-3+7}{4}$ or $y = \dfrac{-3-7}{4}$

$= \dfrac{4}{4}$ $\quad$ $= \dfrac{-10}{4}$

$= 1$ $\quad$ $= -2\frac{1}{2}$

$\sqrt{x}$ cannot be negative

As $y = \dfrac{1}{\sqrt{x}} = 1$, $\sqrt{x} = 1$

$\therefore$ $x = 1$